Course name: **Medical Image Science: Mathematical and Conceptual Foundations**
Course #: **Medical Physics 573 / Biomedical Engineering 573**
Instructors: **Diego Hernando, PhD, Assistant Professor (dhernando@wisc.edu)**
Sean Fain, PhD, Professor (sfain@wisc.edu)
Office hours: **Mondays 10:00AM-11:00AM**
Assistant: **Martin Wagner, PhD**
Session: **Fall 2018**
Credits: **3**
Lectures: **MWF 11:00AM-11:50AM**
Location: **WIMR 1022**
Canvas URL: **https://canvas.wisc.edu/courses/112230**

Instructional mode: **Face-to-face**
Course designations and attributes: **Graduate level, general education**

Course description: This course will cover the mathematical fundamentals required for medical imaging science. These fundamentals include: signal analysis (with an emphasis on Fourier transforms) in one and multiple dimensions, noise in imaging, and image reconstruction. This will be a hands-on course with a combination of theoretical foundations (on the white board) and computational exercises (using a computational environment such as Matlab) on real and simulated datasets. Mathematical concepts will be presented in the context of real-world clinical and research challenges. Upon completion of this course, students should gain a working understanding of a core set of mathematical and computational techniques with multiple applications in medical physics and beyond.

Learning objectives:

Upon completion of this course, students should be able to:

- Understand the concept and utility of signal analysis in one and several dimensions
- Understand and apply convolutions in one and several dimensions
- Understand and apply Fourier transforms in one and multiple dimensions
- Understand and apply the basic properties of the Fourier transform in medical imaging
- Apply signal analysis techniques to understand basic image reconstruction and artifacts
- Know the limitations of the Fourier transform, and understand the properties of alternative signal analysis tools (eg: wavelet transform)
- Understand the stochastic nature of imaging, including various noise distributions
- Calculate image signal-to-noise ratio (SNR) and contrast-to-noise ratio (CNR)
- Understand the connection between signal analysis and stochastic signals with medical physics applications, including in imaging and therapy planning

Prerequisites: Undergraduate calculus, matrix algebra, undergraduate physics, basic statistics. Please see section 0 under course contents below.

Homework: Homework sets will be due every 2-3 weeks, and will include both theoretical derivations and proofs, as well as computational exercises.

Exams: There will be two exams: one midterm and one final exam.

Grading: The midterm exam will count 25% of the final grade. The final exam grade will count for 30%. Homework problems will count for 35%. In-class participation and quizzes will count for the remaining 10%.

Course credit information: This class meets for three 50-minute class period each week over the fall/spring semester and carries the expectation that students will work on course learning activities (reading, writing, problem sets, studying, etc) for about 2 hours out of classroom for every class period. This syllabus includes additional information about meeting times and expectations for student work.

Related courses at UW: This course's materials are continued and complemented in Medical Physics 574 (Fain).

Required texts: No texts are required for this course. Some relevant texts and resources include:

- **Medical Imaging Signals and Systems** by Jerry L. Prince, Jonathan Links, Pearson Education, 2nd Edition.
- **The Fourier Transform & Its Applications** by Ronald Bracewell, McGraw-Hill.
- Additionally, Prof. Barry van Veen's excellent video lectures are available on the **AllSignalProcessing YouTube channel**: https://www.youtube.com/channel/UCooRZ0pxedi179pBe9aXm5A

Academic Integrity:

By enrolling in this course, each student assumes the responsibilities of an active participant in UW-Madison's community of scholars in which everyone's academic work and behavior are held to the highest academic integrity standards. Academic misconduct compromises the integrity of the university. Cheating, fabrication, plagiarism, unauthorized collaboration, and helping others commit these acts are examples of academic misconduct, which can result in disciplinary action. This includes but is not limited to failure on the assignment/course, disciplinary probation, or suspension. Substantial or repeated cases of misconduct will be forwarded to the Office of Student Conduct & Community Standards for additional review. For more information, refer to studentconduct.wiscweb.wisc.edu/academic-integrity/.

Accommodations for Students with Disabilities:

McBurney Disability Resource Center syllabus statement: "The University of Wisconsin-Madison supports the right of all enrolled students to a full and equal educational opportunity. The Americans with Disabilities Act (ADA), Wisconsin State Statute (36.12), and UW-Madison policy (Faculty Document 1071) require that students with disabilities be reasonably accommodated in instruction and campus life. Reasonable accommodations for students with disabilities is a shared faculty and student responsibility. Students are expected to inform faculty [me] of their need for instructional accommodations by the end of the third week of the semester, or as soon as possible after a disability has been incurred or recognized. Faculty [I], will work either directly with the student [you] or in coordination with the McBurney Center to identify and provide reasonable instructional accommodations. Disability information, including instructional accommodations as part of a student's educational record, is confidential and protected under FERPA." http://mcburney.wisc.edu/facstaffother/faculty/syllabus.php

Diversity & Inclusion:

Institutional statement on diversity: "Diversity is a source of strength, creativity, and innovation for UW-Madison. We value the contributions of each person and respect the profound ways their identity, culture, background, experience, status, abilities, and opinion enrich the university community. We commit ourselves to the pursuit of excellence in teaching, research, outreach, and diversity as inextricably linked goals.

The University of Wisconsin-Madison fulfills its public mission by creating a welcoming and inclusive community for people from every background – people who as students, faculty, and staff serve Wisconsin and the world." https://diversity.wisc.edu/

Course contents:

0. Prerequisites: Complex Variables, Matrix Algebra, Basic Statistics	
Theory	***Computation***
1. Complex numbers 2. Vectors and matrices 3. Norms 4. Functions of one and multiple variables 5. Least-squares fitting 6. Eigenvalues and eigenvectors 7. Singular Value Decomposition 8. Basic probability and statistical distributions	• Familiarity with Matlab or other computational tool (eg: Python) • Operating with matrices and vectors • Matrix decompositions • Multi-dimensional arrays and operations

1. Signal Analysis: Fourier and Wavelet Theory and Applications	
Theory	***Computation***

1. Signal analysis basics
2. Linear shift-invariant systems
3. Continuous Fourier Transform
4. Discrete Fourier Transform and Fast Fourier Transform
5. Basic properties of the Fourier transform
    a. Convolution,
    b. Central Slice Theorem,
    c. Symmetry, etc.
6. Fourier Transform in multiple dimensions
7. Sampling in one and multiple dimensions
8. Applications of the Fourier Transform
    a. Convolution
    b. Image reconstruction
9. Short Time Fourier Transform
10. Wavelets in one and multiple dimensions
    a. Motivation
    b. Description
    c. Applications
	• FFT
	• Convolution
	• Convolution examples in therapy planning (dose calculation)
	• Multi-dimensional Fourier
	• Central Slice Theorem examples
	• Filtering
	• Magnitude vs phase information in Fourier space
	• Aliasing and artifacts when representing/reconstructing an image from limited Fourier samples
	• Image compression concepts
	• Wavelet transforms and applications

2. Stochastic Signals and Image Noise	
Theory	***Computation***

1. Motivation for studying stochastic signals
    a. Imaging
    b. Therapy
2. Probability concepts
    a. Random variables, moments, and distributions
3. Autocovariance and power spectrum
    a. Assumptions of noise stability and stationarity
    b. Estimating noise behavior from data
    c. Ergodicity and noise covariance
    d. Role of windowing in spectral estimation
4. Linear systems approach to noise characterization and reduction
    a. Signal and contrast to noise measurement and estimation
    b. Noise and signal detection – Receiver operator characteristic (ROC) analysis
	• Random number generators
	• Monte-Carlo simulation methods
	• Noise propagation through various transformations
	• Calculation of image SNR and CNR
	• Signal detection and ROC analysis exercise
	• Entropy, MSE, and SSIM measures with varying SNR
	• Linear and non-linear filter exercises

c. Image filters for improving signal to noise ratio
    i. Application of 1D and 2D filters
    ii. Adaptive and matched filter approaches
5. Non-linear noise bias and estimation
  a. Noise perception
    i. Entropy and information theory
  b. Measures of bias
    i. Mean square error (MSE)
    ii. Structural similarity index (SSIM)
  c. Non-linear filters for noise reduction
    i. Median filtering
    ii. Statistical (Bayesian) and model-based filters (iterative methods)
    iii. Non-Local Means	

Related courses at other institutions:

UCL: MPHY3893 - Mathematical Methods in Medical Physics (Jem Hebden)
http://www.ucl.ac.uk/medphys/prospective-students/modules/mphy3893

UT: GS02 1183 – Applied Mathematics in Medical Physics
http://www.uthgsbsmedphys.org/GS02-0183/Syllabus%20Math%20Med%20Phys%2009-07-2012.pdf

U Chicago: MPHY 34900 - Mathematics for Medical Physics
http://medicalphysics.uchicago.edu/program/descriptions.html

Duke: MP 530 - Modern Medical Diagnostic Imaging System.
https://medicalphysics.duke.edu/courses

Stanford: EE369C - Medical Image Reconstruction (Pauly)
http://web.stanford.edu/class/ee369c/index.html

USC: EE 592 - Computational Methods for Inverse Problems (Haldar)
https://web-app.usc.edu/soc/syllabus/20173/30794.pdf

Michigan: Bioengineering/Math 464 – Inverse Problems
http://bme.umich.edu/course/biomede-464/

Michigan: EECS 755 - Topics in Signal Processing: Model-based image reconstruction methods (Fessler)
https://web.eecs.umich.edu/~fessler/course/755/index.html